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Nutation of Mars

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PREFACE

The work described in this report was performed by the Systems Division of the Jet Propulsion Laboratory.

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ABSTRACT

The mathematical theory of the nutation of Mars is derived by classical rigid-body dynamics. The effect of nutation is to produce a 26-m maximum horizontal amplitude oscillation (at the surface of Mars) with a period of half a Martian year. This effect should be detectable in the Viking-Lander data.

I. Introduction

The nutation of the Earth results from the action of both the Moon and Sun and is a consequence of the small periodic parts of their gravitational couples (the nonperiodic parts produce the steady precession). Where the satellites of Mars are concerned, their motions are nearly in the equatorial plane, and thus their effect is negligible because of their small masses; consequently, it is sufficient to consider the action of the Sun alone. However, the eccentricity e of the Martian orbit is so much greater than that of the orbit of the Earth that the analytical expression for the solar couple on Mars must be taken to higher order to achieve comparable accuracy. In what follows, all terms in the nutation of Mars of order up to e^2 will be found ($e = 0.0933$ for Mars, as compared with 0.0167 for the Earth).

The principal moments of inertia of Mars will be denoted by A, B, C, and chosen such that $C > A$, and that $A = B$. Numerically for Mars $(C - A)/C = 1/192$ (Ref. 1); the obliquity of the orbital plane in relation to the equator of the planet is $25^\circ\ 12'$; the sidereal rotation period $2\pi/\omega$ is $24^h\ 37^m\ 23^s$; and the orbital period $2\pi/n$ is 687 mean solar days.

II. Theory of Nutation

This theory is given in adequate detail for present purposes in Chapter XX of Ref. 2, and provides the mathematical basis for this report.

In Fig. 1, $OX_oY_oZ_o$ are fixed rectangular axes at the center of Mars, with OX_oY_o the plane of the orbit of the Sun about Mars. The principal moments of inertia (A, A, C) have axes OX, OY, OZ, respectively, as shown, and their positions relative to $OX_oY_oZ_o$ are defined by Eulerian angles θ, ψ, ϕ as shown (Ref. 2 notation adopted). The direction of X_o will be defined later.

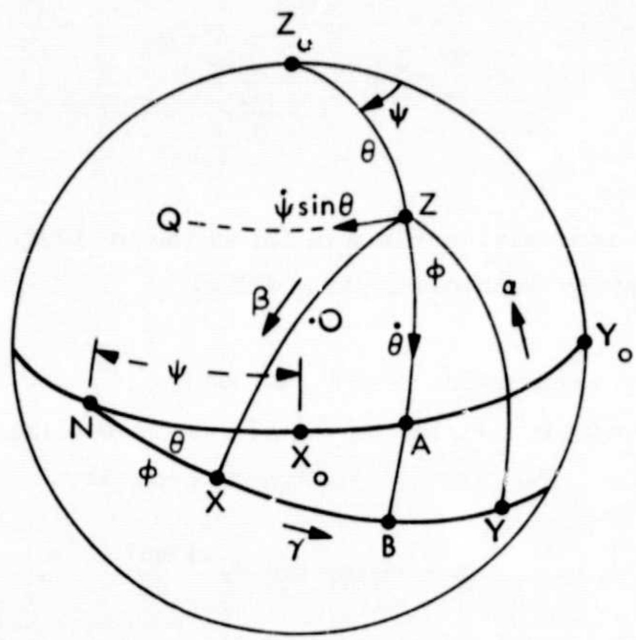

Fig. 1. Mars Geometry

It may readily be shown that the slightly elliptical character of the equator of Mars, recently determined, would give rise to a fourth-order term in the potential U only of order 10^{-6} of that on which the current dynamical calculation rests, and leads to differences in the values of surface measurements of lengths of a few tenths of a millimeter at most. The fifth-order terms are less than 10^{-11} of U itself.

Neglecting higher order effects, the Lagrange potential for determining the couples is shown (Ref. 2, p. 353) to be

$$U = -\frac{3}{2}(C - A)\, n^2 \left(\frac{a}{\rho}\right)^3 \left(\frac{z}{\rho}\right)^2 \tag{1}$$

n = mean motion of Mars about the Sun

a = semimajor axis of orbit about the Sun

ρ = Mars – Sun distance

and the polar equation of the Martian orbit is

$$\frac{a}{\rho} = \frac{1 + e \cos \nu}{1 - e^2} \tag{2}$$

with ν the true anomaly, while z in (1) is the perpendicular distance of the Sun above the Mars equatorial plane OXY.

Under the assumption that $\ddot{\theta}$, $\dot{\theta}^2$ and $\ddot{\psi}$, $\dot{\psi}^2$, $\dot{\theta}\dot{\psi}$ are small compared with $\omega\dot{\psi}$ and $\omega\dot{\theta}$ the equations of motion leading to the nutation-terms (Ref. 2, p. 352) are, to sufficient accuracy,

$$\dot{\psi} = -(C\, \omega \sin \theta)^{-1} \frac{\partial U}{\partial \theta} \tag{3}$$

$$\dot{\theta} = (C\, \omega \sin \theta)^{-1} \frac{\partial U}{\partial \psi} \tag{4}$$

In the steady precession of the planet, the angle ψ increases uniformly with time, so that $\psi = \psi_o + pt$ (where $2\pi/p$ = period of precession), while θ remains constant at a value θ_o, say. It is the small differences from these ψ_o and θ_o in the motion of the point Z that constitute the nutation.

The function U needs to be expressed in terms of the time t and θ, ψ, and other constant angles associated with the orbital motion. But it is of interest that if U is developed in terms of the true anomaly ν, then finite expressions emerge for $\partial U/\partial\theta$, $\partial U/\partial\psi$, and hence for θ, ψ, that would hold for all values of the eccentricity $e < 1$. To show this, it is readily found (Ref. 2, p. 358) that

$$\frac{z}{\rho} = \sin\theta \sin(\nu + w + \psi) \tag{5}$$

where w = the angle subtended (at the center of Mars) from its vernal equinox[1] to the Sun at perihelion. This is the same angle measured between Mars' autumnal equinox[2] and Mars' perihelion – centered at the Sun.

Using the geometry and Ref. 3 for expressions for the orbital angles, we find approximately

$$w = 250.6804^\circ + 0.6343^\circ T \tag{6}$$

where T = Julian centuries since 0^h January 1, 1950. Here w includes a precession value (i. e., takes into account some changes due to ψ) and, further, with ψ being so far indefinite in its zero point, it can be considered small.

We then have

$$U = -\frac{3}{2}(C - A)\, n^2 \left(\frac{a}{\rho}\right)^3 \sin^2\theta \sin^2(\nu + w + \psi) \tag{7}$$

But in the elliptic motion

$$\rho^2 \frac{d\nu}{dt} = h = n\, a^2 \sqrt{1 - e^2} \tag{8}$$

[1]Direction from Mars to Sun at beginning of spring.

[2]Direction from Mars to Sun at beginning of Mars winter.

and hence $d/dt = h\, \rho^{-2}\, d/d\nu$, so that Eqs. (3) and (4) can be transformed to involve differentiation with respect to ν instead of t, thus

$$\frac{d\psi}{d\nu} = -(C\,\omega \sin\theta)^{-1} \frac{\partial U}{\partial \theta} \frac{\rho^2}{h} \tag{9}$$

$$\frac{d\theta}{d\nu} = (C\,\omega \sin\theta)^{-1} \frac{\partial U}{\partial \psi} \frac{\rho^2}{h} \tag{10}$$

Since ρ is independent of θ and ψ, the function $U' = (\rho^2/h)U$ can be used now instead of U, and we have

$$U' = -\frac{3}{4}(C - A)\, n\, (1 - e^2)^{-3/2} \sin^2\theta\, (1 + 3\cos\nu) \left\{1 - \cos(2\nu + 2w)\right\} \tag{11}$$

Thus, the equations of motion take the form

$$\frac{d\psi}{d\nu} = \frac{3}{2}\frac{(C - A)}{C}\frac{n}{\omega}(1 - e^2)^{-3/2} \cos\theta \left\{1 - \cos(2\nu + 2w) + e\cos\nu \right.$$

$$\left. -\frac{1}{2} e \cos(2w + 3\nu) - \frac{1}{2} e \cos(2w + \nu)\right\} \tag{12}$$

and

$$\frac{d\theta}{d\nu} = -\frac{3}{2}\frac{(C - A)}{C}\frac{n}{\omega}(1 - e^2)^{-3/2} \sin\theta \left\{\sin(2w + 2\nu) + \frac{1}{2} e \sin(2w + 3\nu) \right.$$

$$\left. + \frac{1}{2} e \sin(2w + \nu)\right\} \tag{13}$$

Since w increases with time (through ψ) only extremely slowly, it follows that insofar as (1) is adopted as adequate representation of the potential, the values of $\psi(\nu)$ and $\theta(\nu)$ obtained on integration of (12) and (13) are closed expressions valid for all values of $e < 1$. Thus it readily follows, omitting the very slow precessional change of ψ with time, that

$$\psi - \psi_o = \frac{3}{2}\frac{(C - A)}{C}\frac{n}{\omega}(1 - e^2)^{-3/2} \cos\theta \left\{\nu - \frac{1}{2}\sin(2w + 2\nu) + e \sin \nu - \frac{1}{6} e \sin(2w + 3\nu) - \frac{1}{2} e \sin(2w + \nu)\right\} \tag{14}$$

and

$$\theta - \theta_o = \frac{3}{2}\frac{(C - A)}{C}\frac{n}{\omega}(1 - e^2)^{-3/2} \sin\theta \left\{\frac{1}{2}\cos(2w + 2\nu) + \frac{1}{6} e \cos(2w + 3\nu) + \frac{1}{2} e \cos(2w + \nu)\right\} \tag{15}$$

In order to express ψ and θ in terms of the time explicitly, it would be necessary to expand the trigonometrical terms in Eqs. (14) and (15) in terms of the time, which can readily be done by means of standard formulas. But for the purpose, series-expansions in powers of e are necessary and would hold only for sufficiently small values of the eccentricity. In fact, for Mars $e = 0.0933$, and numerical results later obtained herein show that expansion as far as terms in e^2 will give quite sufficient accuracy for any likely application, such as locations of positions on the Martian surface. Alternatively, the function U as given by Eq. (7) can be developed at once in terms of the time as a power series in e, and then use made of Eqs. (3) and (4). Both methods lead, of course, to precisely the same final results.

When the second procedure is adopted, since using M for the mean anomaly $nt + \epsilon - \tilde{\omega}$,

$$\nu = M + 2e \sin M + \frac{5}{4} e^2 \sin 2M$$

it is found that

$$U = -\frac{3}{4}(C - A)\, n^2 \sin^2\theta \left[1 + \frac{3}{2}e^2 - \left(1 - \frac{5}{2}e^2\right) \cos(2M + 2w)\right.$$

$$+ 3e \cos M + \frac{9}{2}e^2 \cos 2M + \frac{1}{2}e \cos(M + 2w) - \frac{7}{2}e \cos(3M + 2w)$$

$$\left. - \frac{17}{2} e^2 \cos(4M + 2w)\right] \qquad (16)$$

Then forming Eqs. (3) and (4) from this expression for U, and integrating with respect to t, it is found that

$$\psi - \psi_o = \frac{3}{2} \frac{(C - A)}{C} \frac{n}{\omega} \cos\theta \left[\left(1 + \frac{3}{2}e^2\right) nt\right.$$

$$- \left(\frac{1}{2} - \frac{5}{4}e^2\right) \sin(2M + 2w) + 3e \sin M + \frac{1}{2} e \sin(M + 2w)$$

$$\left. - \frac{7}{6} e \sin(3M + 2w) + \frac{9}{8} e^2 \sin 2M - \frac{17}{8} e^2 \sin(4M + 2w)\right]$$

(17)

and

$$\theta - \theta_o = \frac{3}{2} \frac{(C - A)}{C} \frac{n}{\omega} \sin\theta \left[\left(\frac{1}{2} - \frac{5}{4}e^2\right) \cos(2M + 2w)\right.$$

$$- \frac{1}{2} e \cos(M + 2w) + \frac{7}{6} e \cos(3M + 2w)$$

$$\left. + \frac{17}{8} e^2 \cos(4M + 2w)\right] \qquad (18)$$

We now choose X_o, so far an arbitrary direction, such that $\psi_o = 0$ at the chosen epoch. Thus ψ is always small and can be ignored in the trigonometric arguments.

The term proportional to nt in Eq. (17) represents the steady precession,[3] and if we write $\psi = \psi_o + pt +$ nutation (in ψ), the numerical values for the coefficients of the nutational terms in ψ and θ are found to be (in seconds of arc) as follows

$$\psi = \psi_o + pt - 1''.0715 \sin (2M + 2w) + 0''.6133 \sin M$$

$$+ 0''.1022 \sin (M + 2w) - 0''.2385 \sin (3M + 2w)$$

$$+ 0''.0215 \sin 2M - 0''.0405 \sin (4M + 2w) \tag{19}$$

$$\theta = \theta_o + 0''.5042 \cos (2M + 2w) - 0''.0481 \cos (M + 2w)$$

$$+ 0''.1089 \cos (3M + 2w) + 0''.0185 \cos (4M + 2w) \tag{20}$$

It is seen that the largest coefficients are those of the terms in 2M and thus have a period of half the Martian year. The greatest semi-amplitudes of $\psi \sin \theta$ and θ are seen to amount in sum to about 2'' in ψ at the equator, and 0''.6 in θ. These would correspond, respectively, to displacements of about 26 m and 9 m of the surface relative to a steadily precessing Mars.

[Note added in proof, August 1979: Analysis of the Viking-Lander data has confirmed oscillations of the foregoing order of magnitude. The relevant data and analysis will be published in due course.]

[3] The value of precession for this development, 7''.415/yr, follows from (C - A)/C = 0.00524 (Ref. 1).

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